Structural Dynamics Assignment

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Contents

Introduction and description

During the semester, in the subject of Structural Dynamics, some practical examples were studied, both in the tutorial classes and the regular classes. The students were able to identify and get a glimpse of the usage of theories, calculation algorithms and software used in dynamic analysis by engineers in the industry and research related work.

The objective of this assignment is to make a more thorough analysis of a dynamic system, using the techniques taught throughout the semester.

Structural system description

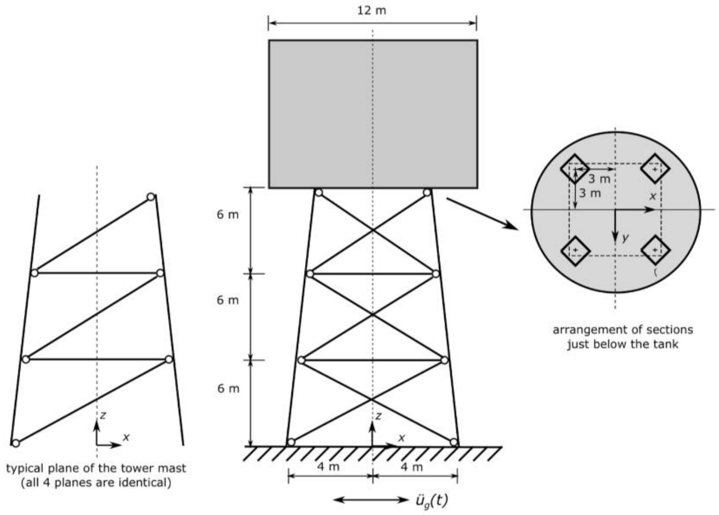

Figure 1 - Water tower system

The water tower system, Figure 1 - Water tower system*Figure 1*, is a 3-dimensional girder that carries a water tank, filled completely and with rigid walls. Its volume is V = 905m^3. The supports of the girder are fixed, and the load-case to be studied is an earthquake applied on the x direction, as shown.

The dimensions of the water tower which were provided are given on the figure above, and the information about cross-sections and the liquid that fills the tank are provided on *Table 1*.

Column quadratic hollow ***a x t* [mm]**	320 x 12,5
Diagonal member circular hollow ***D x t* [mm]**	70 x 10
Horizontal member circular hollow ***D x t* [mm]**	48,3 x 8,8
Mass density of liquid filling **ρ [kg/m³]**	1000

Table 1 - Cross section and liquid information

Analysis description, theoretical background

Part 1: Preliminary hand-calculations

The analysis is divided into 2 parts: the first part consists of simplifying the system into a single-degree-of-freedom oscillator, like the one on Figure 2, with all the mass (***m***) being concentrated on the water tank and the stiffness (***k***) on the girder.

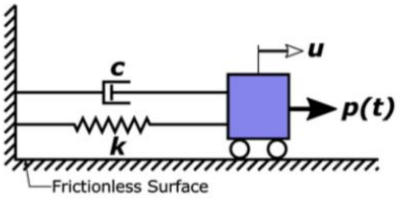

Figure 2 - SDOF oscillator

Using this simplification, it is required to calculate the fundamental natural frequency of the water tower. This is done by modelling the geometry on Ansys, applying an arbitrary load to the top of the tower and, with the displacement given on the program, computing the stiffness using Hooke's law (*Equation 1*). From that, the natural frequency can be obtained by *Equation 2*. The mass is simply obtained with the geometry and liquid material properties given, by *Equation 3*.

$$F\ [N] = k \times x$$

Equation 1 - Hooke's law

$$\omega \,[\frac{rad}{s}] = \sqrt{\frac{k}{m}}$$

Equation 2 - Fundamental natural frequency

$$m\,[kg] = V \times \rho$$

Equation 3 - Mass

Also, the results of a free vibration test, performed in the structure to determine its damping ratio, are given. It has been observed that the amplitude of the displacement decays to 50% of its initial value with 4 complete cycles of vibration.

This way, utilizing the technique of the logarithmic decrement (*Equation 4*) to determine the damping ratio in an experimental way (through the free vibration test), more information about the damping properties in the system can be obtained.

$$\delta = \ln\frac{u_i}{u_{i+1}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Equation 4 - Logarithmic decrement

Since we can consider ζ to be small, it can be approximated by *Equation 5*.

$$\zeta = \frac{1}{2\pi j} \cdot \ln\frac{u_1}{u_j}$$

Equation 5 - Damping ratio for small damping

The calculation of the maximum bending moment at the bottom of the water tower, using the SDOF case, is also a requirement. For this purpose, the soil and location data are provided on *Table 2*.

Location	Albstadt
Ground type	C
Importance class	IV

Table 2 - Soil and location

Using the DIN EN 1998-1/NA the following equations compute the horizontal response spectrum, after that the base shear force and finally the bending moment at the base can be derived. $S_e(T)$ is in m/s^2.

$$\begin{cases} 0 \leq T \leq T_B : \; S_e(T) = a_{gR} \cdot \gamma_1 \cdot S \cdot [1 + \frac{T}{T_B} \cdot (\eta \cdot 2{,}5 - 1)] \\ T_B \leq T \leq T_C : \; S_e(T) = a_{gR} \cdot \gamma_1 \cdot S \cdot \eta \cdot 2{,}5 \\ T_C \leq T \leq T_D : \; S_e(T) = a_{gR} \cdot \gamma_1 \cdot S \cdot \eta \cdot 2{,}5 \cdot [\frac{T_C}{T}] \\ T_D \leq T \leq 4s : \; S_e(T) = a_{gR} \cdot \gamma_1 \cdot S \cdot \eta \cdot 2{,}5 \cdot [\frac{T_C T_D}{T^2}] \end{cases}$$

Equation 6 - Horizontal response spectrum (according to DIN EN 1998-1/NA)

$$F_b[N] = S_e(T_1) \times m \times \lambda$$

Equation 7 - Base shear force (according to DIN EN 1998-1/NA)

The damping correction factor (η) is calculated by *Equation 8* and is dimensionless.

$$\eta = \sqrt{10/(5 + \zeta)} \geq 0{,}55$$

Equation 8 - Damping correction factor (according to DIN EN 1998-1/NA)

The variables γ_1 and λ in these formulas can be found inside the code text itself. For the tower, importance class and its size (only one story high) are respectively 1,4 and 1,0. γ_1 refers to the importance of the building for public safety and civil protection in case of seismic activity and λ is a correction factor that accounts for the fact that in buildings with at least 3 stories and translational degrees of freedom in each horizontal direction, the effective modal mass of the 1st mode is smaller than the total building mass.

The other variable values to calculate the horizontal response spectrum need to be taken from the German National Annex for the ground and seismic characteristics in the Albstadt region, as stated in the European Standard. *Table 3* summarizes and groups the values used in this analysis.

The article Die Neue Erdbebennorm DIN 4149, by Prof. Dr.-Ing. A. Ötes of the Universität Dortmund was used as a source for the values hereby presented.

T_C (s)	0,3
T_D (s)	2,0
S	1,5
a_g (m/s²)	0,8

Table 3 - Parameters to be used in the elastic response spectra (according to the German Annex)

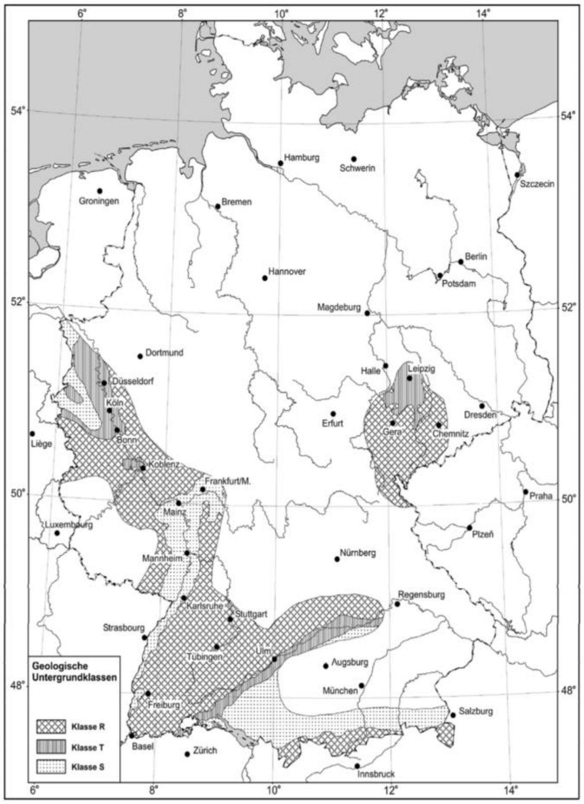

Figure 3 - Deep geology information (according to the German Annex)

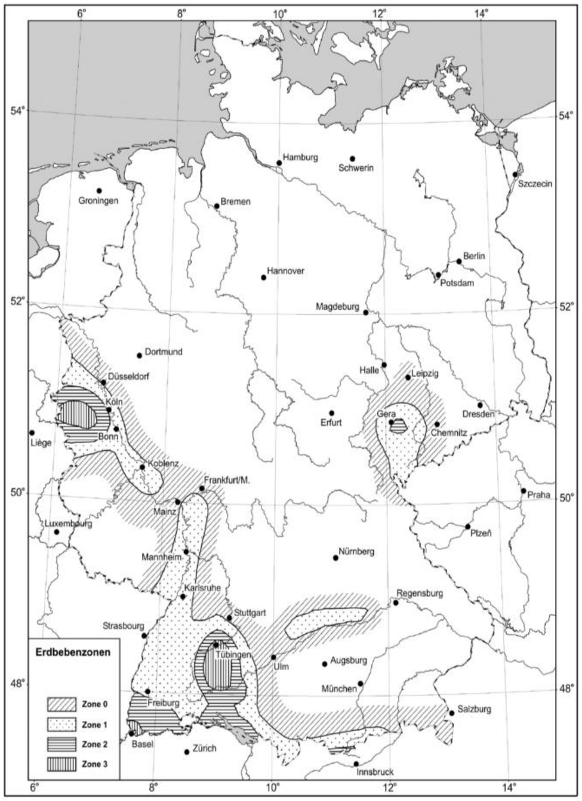

Figure 4 - Seismic activity information (according to the German Annex)

In both cases, Albstadt can be categorized in the same zone as Tübingen.

Part 2: Modelling and verification

For Part 2, the same ANSYS model used to calculate the stiffness of the tower can be used to obtain, computationally, the first two distinct natural frequencies and mode shapes of the system with a modal analysis, verify the free vibration test provided with a transient analysis and calculate the maximum bending moment at the base using the response spectrum analysis.

This part should be done in a more detailed and careful way, so the results are closer to reality than the previously done hand calculations. In the other hand, Part 1 serves as a guide and the values should be close to each other. In this sense, material elastic and inertial information (Young’s modulus, Poisson’s ratio and density) are added to the girder beams, as shown in *Table 4*.

E [N/m²]	210 x 10^9
ν	0,28
ρ [kg/m³]	7850

Table 4 - Material elastic and inertial information

The model generated on Part 1 must contain all the geometry and material information. The connections and mesh are generated automatically by the software.

The time step chosen for the transient analysis should be a fraction of the highest between the two first distinct eigenfrequencies. A reasonable value for it can be obtained by dividing the final time by a value from 20 to 50. Damping is set, in this kind of analysis, from the calculated damping ratio (*Equation 5*), also related to the natural frequency used to choose the time step, as the damping is given by the stiffness coefficient calculated automatically on Ansys with these 2 parameters.

For the response spectrum analysis, the same spectrum as in Part 1 is used. Modal analysis is a requirement for its realization.

Results, evaluations and conclusions

Model and mesh

Two plots of the generated model and mesh are shown, respectively, below in *Figure 5 and Figure 6*, and *Figure 7 and Figure 7*, where the elements are already created. The x-, y- and z- axes are the same as in *Figure 1*.

A, B, C and D are a representation of the distributed mass, which is simply the weight load (of the water tank) applied on the system. This is reflected on the model as 4 rigid beams connecting the top part of the girder, each them with ¼ of the total mass calculated. Different cross-sections are applied, as provided in *Table 1*.

The mesh created by Ansys contains 508 nodes and 264 elements.

Total mass (m) and the mass of each beam ($m_{A,B,C,D}$) are calculated below:

$$\mathrm{m} = \mathrm{V} \times \rho = 905\mathrm{m}^3 \times 1000\frac{\mathrm{kg}}{\mathrm{m}^3} = 9{,}05 \cdot 10^5\ \mathrm{kg} \rightarrow \mathrm{m}_{\mathrm{A,B,C,D}} = \frac{\mathrm{m}}{4} \cong 2{,}263 \cdot 10^5\ \mathrm{kg}$$

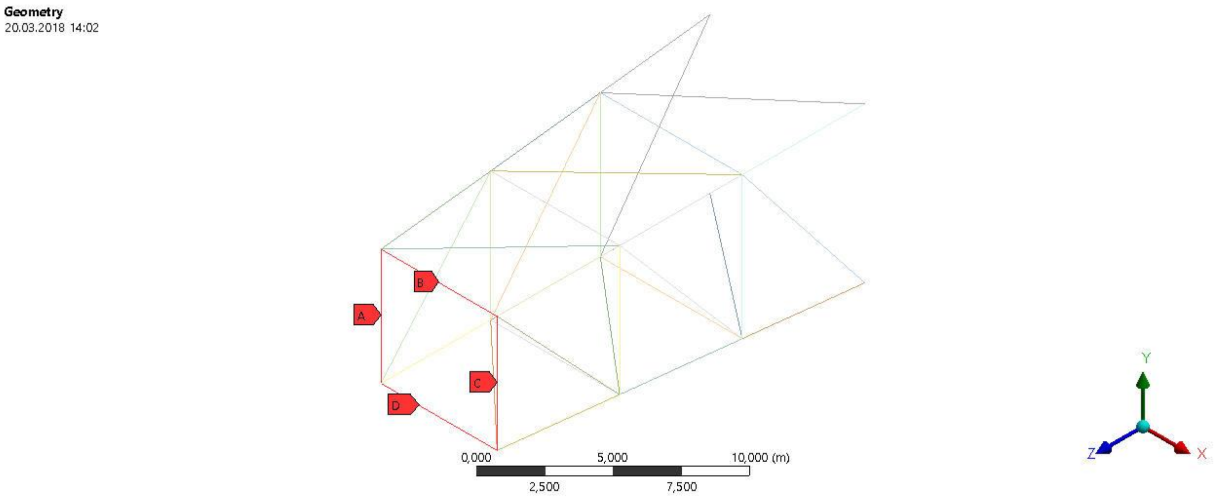

Figure 5 - Model

Geometry
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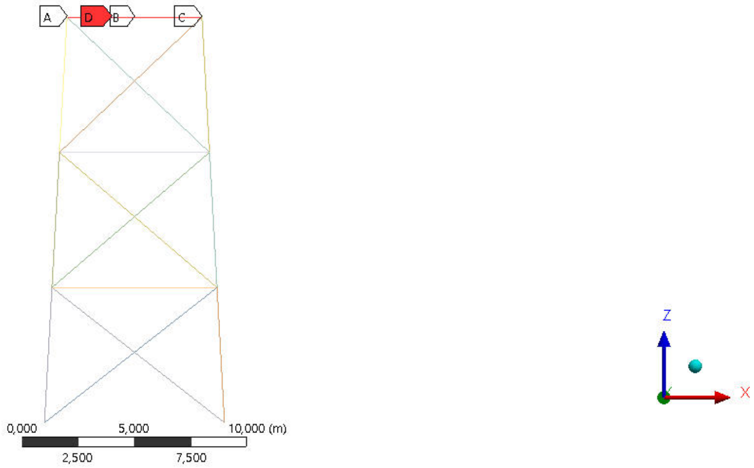

Figure 6 - Model

Geometry
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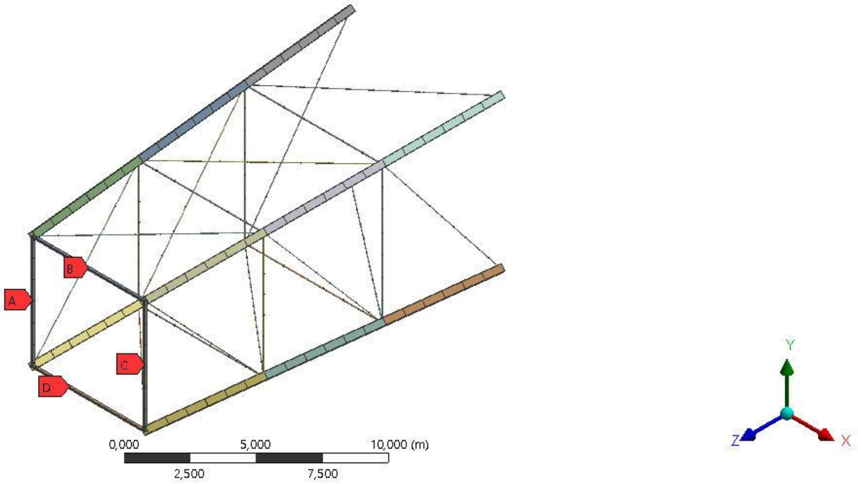

Figure 7 - Mesh

Geometry
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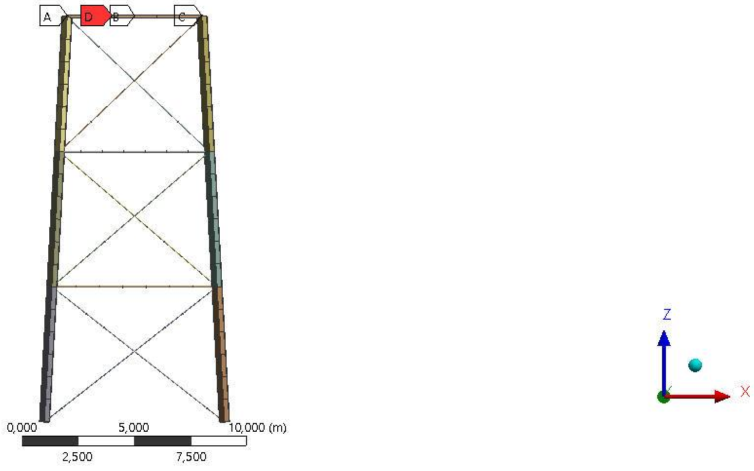

Figure 8 - Mesh

Natural frequencies and natural mode shapes

Applying a lateral load of $1 \cdot 10^6$ N across the whole upper left bar (which contains the concentrated mass indicated as A on *Figures 5, 6, 7 and 8*), gives the deformation needed to simplify the system into a SDOF oscillator and estimate its stiffness, calculated below using the displacement (x) provided in *Figure 9*. Material properties are not considered on this part.

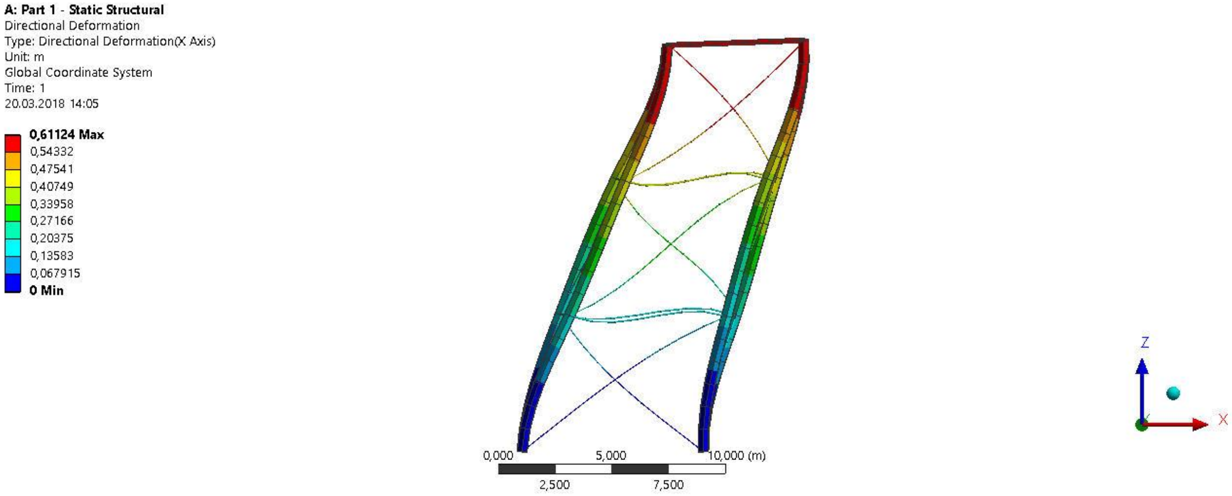

Figure 9 - Deformation used for SDOF simplification

$$\mathrm{F} = \mathrm{k} \times \mathrm{x} \rightarrow \mathrm{k} = \frac{\mathrm{F}}{\mathrm{x}} = \frac{1000000 \cdot 6}{0{,}611} \cong 9{,}82 \cdot 10^6\ \frac{\mathrm{N}}{\mathrm{m}}$$

From this, the natural frequency can easily be obtained:

$$\omega = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}} = \sqrt{\frac{9{,}82 \cdot 10^6}{9{,}05 \cdot 10^5}} = 3{,}294\ \frac{\mathrm{rad}}{s} \rightarrow \mathrm{f} = \frac{\omega}{2\pi} = 0{,}524\ \mathrm{Hz}$$

By creating a modal analysis on Ansys, with the system, the first two distinct eigenmodes can be obtained and are shown on *Figures 10 and 11*, the first three eigenvectors are summarized on *Table 5*. Material properties described on *Table 4* are now turned on.

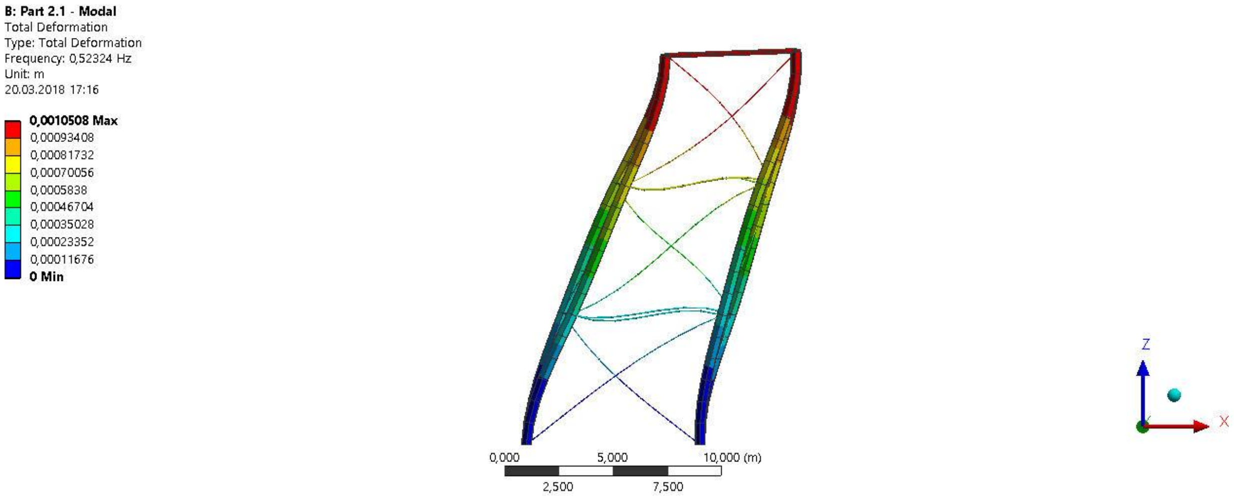

Figure 10 - 1st natural mode shape

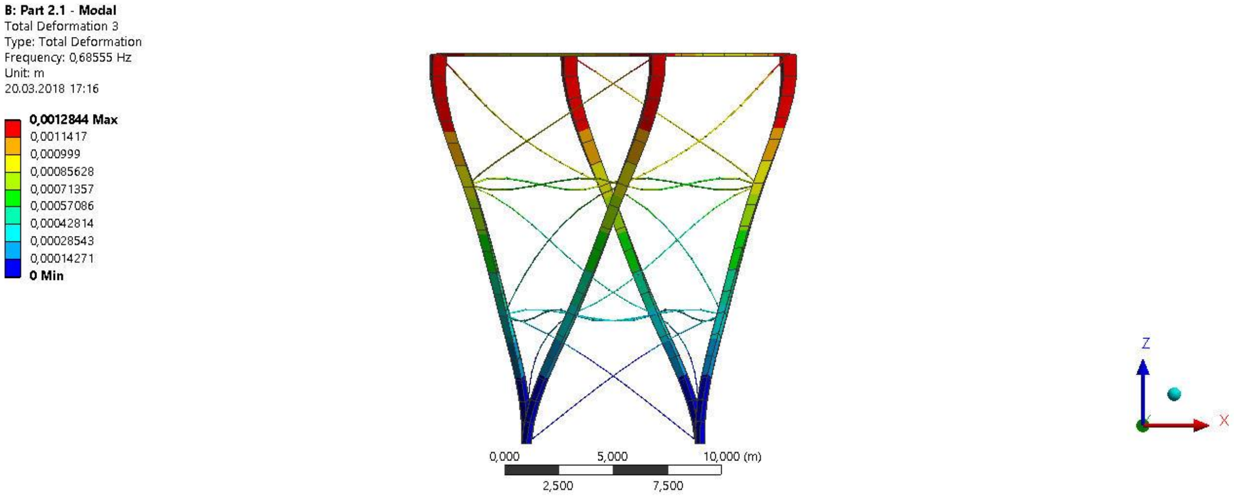

Figure 11 - 3rd natural mode shape

	Mode 1	**Mode 2**	**Mode 3**
f [Hz]	0,523	0,523	0,686
T [s]	1,912	1,912	1,458

Table 5 - Three first natural frequencies and periods of vibration

The first two natural frequencies have the same value, but they create motion on 2 different axes, x- for the 1st mode shape and y- for the 2nd, this makes the 3rd the other distinct eigenvector.

The result given by Ansys is only different by 0,001 Hz from the one calculated before. This can be due to the material properties, that were previously unassigned, or because of the approximation made to transform the structure into a 2D SDOF system.

Free vibration test

With the results from the free vibration test, the damping ratio can be calculated:

$$\zeta = \frac{1}{2\pi j} \cdot \ln \frac{u_1}{u_j} = \frac{1}{2\pi \cdot 4} \cdot \ln \frac{u_1}{0{,}5 \cdot u_1} = \frac{1}{2\pi \cdot 4} \cdot \ln 2 \cong 0{,}028$$

This, indeed, confirms that the damping is small, with ζ=2,8%.

On Ansys, the FVT is verified with the model through a transient analysis, in which an initial displacement of 1m on the positive x- axis is given, at the top of the tower (same place as the previously assigned force load), and the time step applied accordingly to the critical mode desired. That is the 2nd distinct natural mode, in this case, because it has the highest frequency and consequently the lowest period of vibration (T_C). The damping ratio is also set accordingly to the same critical mode. The program uses the damping ratio and the frequency specified to calculate the correspondent stiffness coefficient.

In this way, considering 7 cycles of vibration (j) with 5 sub steps (n) each, the time step (t_s) is defined as (10,171 is the final time. It is divided by a value between 20 and 50):

$$t_s = \frac{T_C \cdot j}{j \cdot n} = \frac{T_3 \cdot 7}{7 \cdot 5} = \frac{10{,}171}{35} \cong 0{,}292\ \mathrm{s}$$

In this kind of analysis, on Ansys, the load needs to be applied as a time step, so an initial 0,001 step containing the displacement given is defined as time step 1, with 3 sub steps so the velocity is zero at the end of this period, and the rest as time step 2 with 35 sub steps.

The result is quite like what was expected and is summarized by *Figures 12 and 13* and *Table 6*.

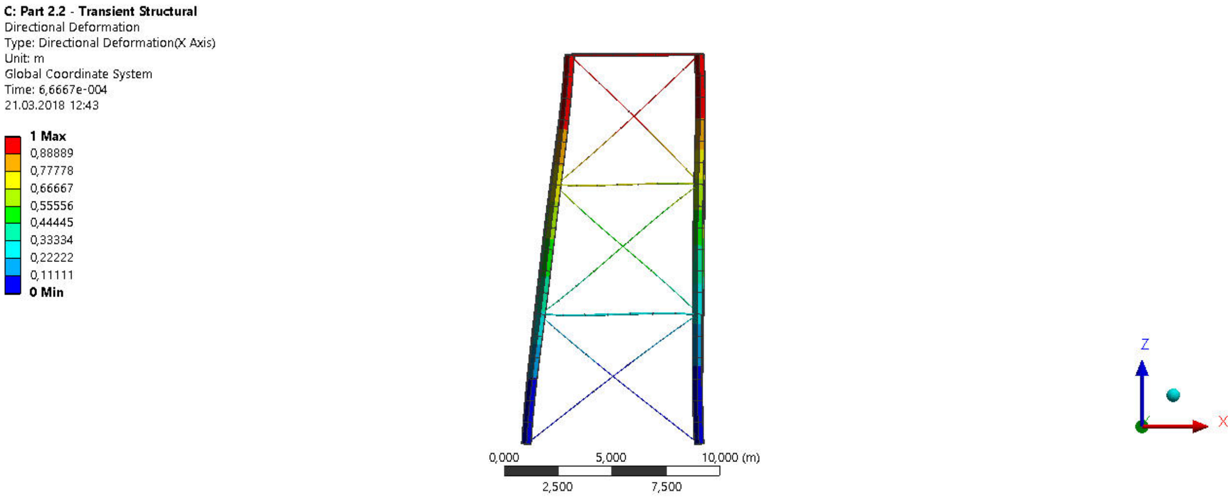

Figure 12 - Initial displacement

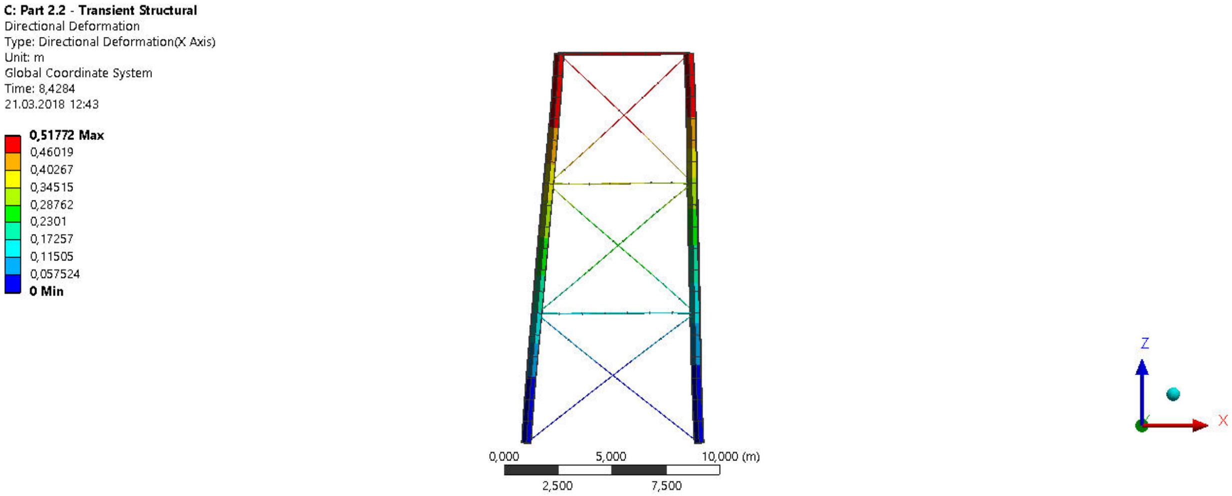

Figure 13 - Displacement at the end of 4 cycles

Steps 3 and 32 are highlighted on the table because they refer to *Figures 12 and 13* temporal snapshots in the analysis, respectively. One being the initial displacement (initial amplitude) and the other the 4th cycle's amplitude.

The small difference of approximately 0,018m can be observed possibly due to errors on the formula that approximates the damping ratio used in the analysis, or on the calculation of the frequency, through the modal analysis.

Step	Minimum [m]	Maximum [m]
1	0	1
2	0	1
3	0	1
4	0	0,78817
5	0	0,1741
6	-0,5422	0
7	-0,84319	0
8	-0,53534	0
9	0	0,14076
10	0	0,69175
11	0	0,72988
12	0	0,2491
13	-0,3902	0
14	-0,72836	0
15	-0,53623	0
16	0	2,88E-02
17	0	0,55196
18	0	0,66336
19	0	0,29952
20	-0,26285	0
21	-0,61723	0
22	-0,51996	0
23	-5,73E-02	0
24	0	0,42827
25	0	0,59134
26	0	0,32845
27	-0,15752	0
28	-0,51308	0
29	-0,4909	0
30	-0,12112	0
31	0	0,32048
32	0	0,51772
33	0	0,33996
34	-7,23E-02	0
35	-0,41747	0
36	-0,45314	0
37	-0,16582	0
38	0	0,22821

Table 6 - Transient analysis result

Response spectrum

Since, T=1,908s for the natural mode on the SDOF system, and using the values given on *Table 3*, *Equation 3* is simplified to its 3rd part, only:

$$\mathrm{T_C} \leq \mathrm{T} \leq \mathrm{T_D}:\ \mathrm{S_e(T)} = \mathrm{a_{gR}} \cdot \gamma_1 \cdot \mathrm{S} \cdot \eta \cdot 2{,}5 \cdot \left[\frac{\mathrm{T_C}}{\mathrm{T}}\right] \rightarrow \mathrm{S_e}(1{,}908) = 0{,}8 \cdot 1{,}4 \cdot 1{,}5 \cdot 1{,}41 \cdot 2{,}5 \cdot \left[\frac{0{,}3}{1{,}908}\right] \cong 0{,}621$$

γ is 1,4 because the building's class of importance is IV. η is calculated with the damping ratio:

$$\eta = \sqrt{10/(5+\zeta)} = \sqrt{10/(5+0{,}028)} \cong 1{,}41 \geq 0{,}55$$

From this, the base shear force and, consequently, the moment at the base are calculated by:

$$\mathrm{F_b} = \mathrm{S_e(T_1)} \cdot \mathrm{m} \cdot \lambda = 0{,}621 \cdot 9{,}05 \cdot 10^5 \cdot 1{,}0 \cong 5{,}621 \cdot 10^5\ \mathrm{N} \rightarrow \mathrm{M} = \mathrm{F_b} \cdot \mathrm{h} = 5{,}621 \cdot 10^5 \cdot 18 \cong 1{,}012 \cdot 10^7\ \mathrm{N \cdot m}$$

With the same response spectrum, on a response spectrum analysis on Ansys, a similar result can be obtained, proving that the algorithm used for the initial calculation is correct.

Small deviations can happen because of the SDOF simplification and because Ansys uses better methods to apply the response spectrum load case.

Results of the analysis made on the program and with the initial calculation are summarized on *Table 7*.

	Base shear force [N]	**Moment on base [N·m]**
Initial calculation	$5{,}621 \cdot 10^5$	$1{,}012 \cdot 10^7$
Ansys analysis	$5{,}660 \cdot 10^5$	$1{,}015 \cdot 10^7$
Deviations	0,7%	0,3%

Table 7 - Results for response spectrum analysis

Conclusion and further improvements

As previously stated, this assignment was a good exercise of the material and content learned throughout the semester. To further improve the model created, and make it more real, would be hard in a controlled classroom situation, where the time is limited, and the objectives are more pedagogical. Only a real structure analysis could require a more in-depth investigation of peculiarities and abstraction power to model and simplify it, starting from the previous step (since this model was already simplified, from the beginning).

This doesn't mean that the model created was perfect and couldn't be improved. The reason for deviations of the values acquired with the model and the ones calculated with the SDOF oscillator simplification are summarized on each subpart of the results. Further improvements on the analysis would be welcome and here are some suggestions:

1. Model the geometry considering the presence of the water tank, not just creating beams that represent its weight on the structure;
2. Consider nonlinear and inelastic situations;
3. Consider more natural mode shapes;
4. Get better damping information to use on the transient analysis;
5. Use the design response spectrum to consider non-linear effects (making the response spectrum less conservative).

Apart from all this, the results of the model were very close to the SDOF oscillator simplification, showing that, for simple cases, making this abstraction, in practice, is valid and gives a very good idea of the true values in the system.